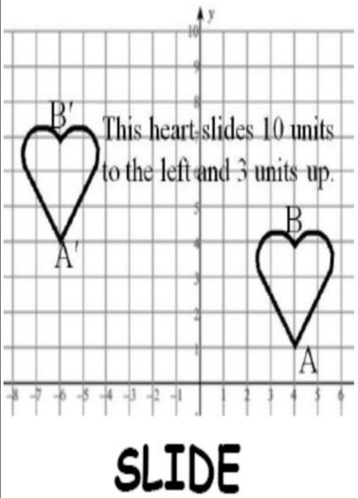
Math 8 Georgia Milestones Review Sheet

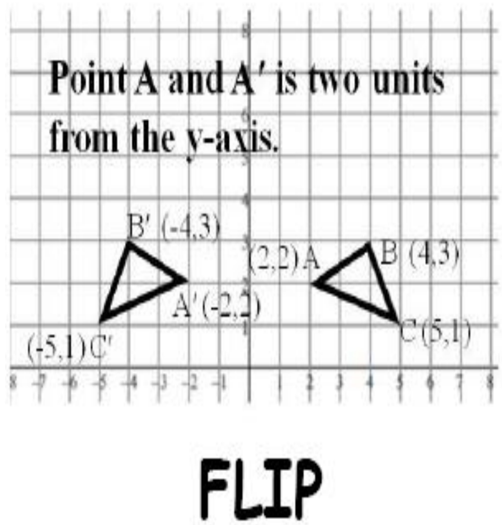
Unit 1: Transformations, Congruence, and Similarity

Transformations…

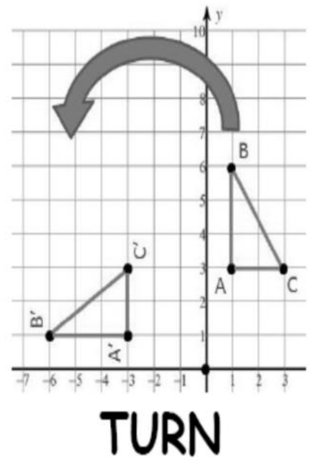
Translation - "slides" each point of a figure the same distance in the same direction without changing its size or shape and without turning it or flipping it.



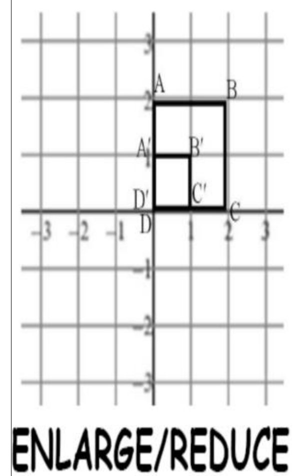
Reflection - "flips" a figure over a mirror or reflection line; An object and its reflection have the same shape and size, but the figures face in opposite directions.



Rotation - turns a figure about a fixed point at a given angle and a given direction; An object and its rotation are the same shape and size, but the figures may be turned in different directions.



Dilation – proportionally changes the size of an object (by shrinking or stretching), but not the shape.



Translations Rules:

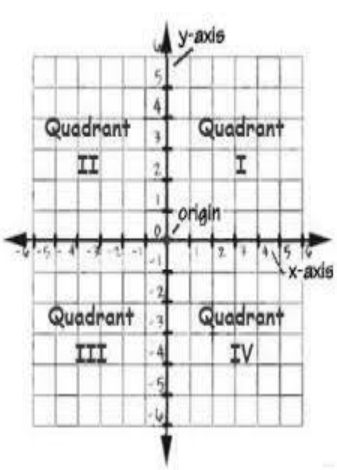
A positive integer describes a translation right or up on a coordinate plane.

A negative integer describes a translation left or down on a coordinate plane.

A movement left or right is on the x-axis.

A movement up or down is on the y-axis.

Coordiante Plane:



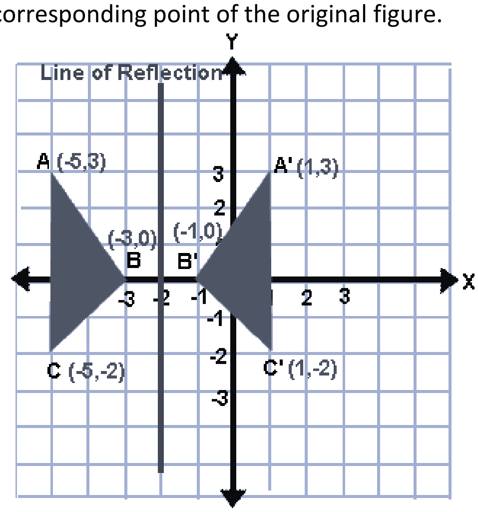
Reflection Rules:

Reflect a figure over the x-axis- when reflecting over the x-axis, change the y-coordinates to their opposites. **(x, -y)**

Reflect a figure over the y-axis- when reflecting over the y-axis, change the x-coordinates to their opposites. **(-x, y)**

Reflection over any line

Each point of a reflected image is the same distance from the line of reflection as the corresponding point of the original figure.



Notice how each point of the original figure and its image are the same distance away from the line of reflection (which can be easily counted in this diagram since the line of reflection is vertical).

Rotation Rules:

90 degree clockwise rotation around the origin (0,0), use: (y, -x)

180 degree rotation around the origin (0,0), use: (-x, -y)

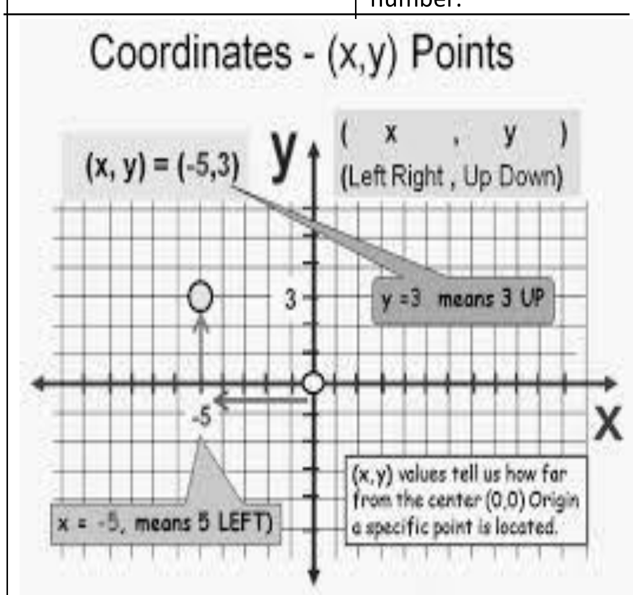
270 degree clockwise rotation around the origin (0,0), use: (-y, x)

Dilation Rules:

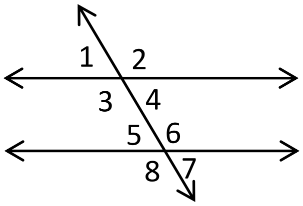
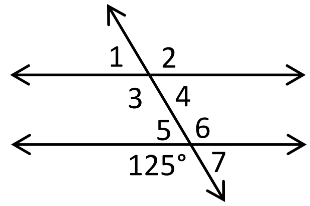
To dilate a figure, always MULTIPLY the coordinates of each of its points by the percent of dilation.

First change the percent to a decimal (move the decimal point TWO places to the LEFT.

Next, multiply each of the coordinates by that number.



Angles…

Relationships		Measurements	
	**Alternate Interior** ∠3 & ∠6 **Alternate Exterior** ∠2 & ∠8 **Consecutive Interior** ∠4 & ∠6 **Corresponding** ∠1 & ∠5 **Vertical** ∠2 & ∠3 **Adjacent** ∠7 & ∠8		m∠**1** =55° m∠**2** = 125° m∠**3** = 125° m∠**4** = 55° m∠**5** = 55° m∠**6** = 125° m∠**7** = 55°
Supplementary angles = 180^0 **Complementary angles** = 90^0 **Linear Pair:** Two angles that are adjacent (share a leg) and supplementary (add up to 180^0)	**Corresponding angles** = on the same side of the transversal (one angle is an interior and one is an exterior) **Vertical angles** are opposite of each other and have the same measurement; are congruent (≅)	**Alternate interior angles** are opposite each other and inside two parallel lines which are cut by a transversal; have the same measurement; are congruent(≅)	**Alternate exterior angles** are opposite each other and outside two parallel lines which are cut by a transversal; have the same measurement; are congruent(≅)

Unit 2: Exponents and Equations

Estimating Radicals:	Scientific Notation:	Multiplying:	Dividing:
1) Draw a number line 2) Find the closest perfect squares – one smaller and one larger 3) Eliminate answer choices	3,420,000 = **3.42×10^6** .00000000986 = **9.86×10^9** 1. Place decimal behind first non 0 number 2. Multiply by 10 3. Count spaces new to old (exponent) Left = negative and right = positive	$(2.3 \times 10^5)(1.4 \times 10^2)$ 1) 2.3 x 1.4 = **3.22** 2) $10^5 \times 10^2 = \mathbf{10^7}$ 3) **3.22×10^7**	$(6 \times 10^8) \div (2 \times 10^2)$ 1) 6 ÷ 2 = **3** 2) $10^8 \div 10^2 = \mathbf{10^6}$ 3) **3×10^6**

Square Roots

Know perfect square roots to 25.	Know perfect cube roots to 10.	Know:	Know:
$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$ $5^2 = 25$ $6^2 = 36$ $7^2 = 49$ $8^2 = 64$ $9^2 = 81$ $10^2 = 100$ $11^2 = 121$ $12^2 = 144$ $13^2 = 169$ $14^2 = 196$ $15^2 = 225$ $16^2 = 256$ $17^2 = 289$ $18^2 = 324$ $19^2 = 361$ $20^2 = 400$ $21^2 = 441$ $22^2 = 484$ $23^2 = 529$ $24^2 = 576$ $25^2 = 625$	$1^3 = 1$ $\sqrt[3]{1} = 1$ $2^3 = 8$ $\sqrt[3]{8} = 2$ $3^3 = 27$ $\sqrt[3]{27} = 3$ $4^3 = 64$ $\sqrt[3]{64} = 4$ $5^3 = 125$ $\sqrt[3]{125} = 5$ $6^3 = 216$ $\sqrt[3]{216} = 6$ $7^3 = 343$ $\sqrt[3]{343} = 7$ $8^3 = 512$ $\sqrt[3]{512} = 8$ $9^3 = 729$ $\sqrt[3]{729} = 9$ $10^3 = 1000$ $\sqrt[3]{1000} = 10$	$\sqrt{0} = 0$ $\sqrt{}$ = radical	$1^0 = 1$ $55^0 = 1$ $100^0 = 1$ Anything to the raised to the zero power is one.

Rational and Irrational Numbers

Rational Numbers	Irrational Numbers
Is a number that can be expressed as a fraction or ratio. The numerator and the denominator of the fraction are both integers. When the fraction is divided out, it becomes a terminating or repeating decimal. **Examples:** 2/3 0.25 ½ 0.666	Cannot be expressed as a fraction. Irrational numbers cannot be represented as terminating or repeating decimals. Irrational numbers are non-terminating, non-repeating decimals. **Example:** π= 3.141592654….. $\sqrt{2}$= 1.414213562….. 0.1211211121112

Exponent Rules – must have the same base!! Keep the base!!

Multiplying:	Dividing:	Power to a Power:	Negatives:
add exponents	subtract exponents	multiply exponents	flip to become positive
$4^2 \times 4^6 = 4^8$	$\frac{6^8}{6^5} = 6^3$	$(3^5)^2 = 3^{10}$	$2^{-6} = \frac{1}{2^6}$
			***Does not apply to scientific notation*

Properties

Commutative Property of Addition
Changing the *order* of the addends does not change the sum.

a + b = b + a
5 + 9 = 9 + 5
14 = 14

Think "order"

Commutative Property of Multiplication
Changing the *order* of the factors does not change the product.

a x b = b x a
3 x 8 = 8 x 3
24 = 24

Associative Property of Addition
Changing the *grouping* of the addends does not change the sum.

(a + b) + c = a +(b + c)
(1 + 4) + 7 = 1 + (4 + 7)
5 + 7 = 1 + 11
12 = 12

Think "grouping"

Associative Property of Multiplication
Changing the *grouping* of the factors does not change the product.

(a x b) x c =a x (b x c)
(6 x 5) x 2 = 6 x (5 x 2)
30 x 2 = 6 x 10
60 = 60

Identity Property of Addition
The sum of zero and a number is that number.

a + 0 = a
0 + a = a

89 + 0 = 89
0 + 89 = 89

Think "same"

Identity Property of Multiplication
The product of one and a number is that number.

1 x a = a   a x 1 = a

1 x 8 = 8   8 x 1 = 8

Zero Property of Multiplication
The product of zero and a number is zero.

0 x a = 0
a x 0 = 0

0 x 33 = 0
33 x 0 = 0

Think "0 product"

Order of Operations

Step 1: Complete the operation inside of the parentheses first.

Step 2: Complete any exponents.

Step 3: Multiply & Divide IN ORDER from LEFT to RIGHT.

Step 4: Add & Subtract IN ORDER from LEFT to RIGHT.

Example:
48 ÷ 8 + 6(4 + 2) - 15

Step 1: 6(4 + 2) = 6(6) = 36
Next Line: 48 ÷ 8 + 36 - 15
Step 2: 48 ÷ 8 = 6
Next Line: 6 + 36 - 15
Step 3: 6 + 36 = 42
Next Line: 42 - 15 = 27

Inverse Property of Addition

Adding the opposite (additive inverse) of a number to the number will give you a sum zero.

Example:
a + (-a) = 0
(-a) + a = 0

14 + (-14) = 0
(-14) + 14 = 0

Inverse Property of Multiplication

If you multiply a number by its reciprocal (multiplicative inverse) the product is 1.

Example:
a x 1/a = 1
1/a x a = 1

9 x 1/9 = 1
1/9 x 9 = 1

Distributive Property

Multiply a sum by multiplying each addend separately and then add the products.

5(x + 2) = 5(x) + 5(2)
= 5x + 10

Can ***NOT* combine unlike terms

Like terms

Terms whose variables (and their <u>exponents</u> such as the 2 in x^2) are the same. In other words, terms that are "like" each other.
Example:

7x x -2x

Are all like terms because the variables are all x

Unlike Terms

Example:
$-3xy$ $-3y$ $12y^2$

these are all unlike terms (xy, y and y^2 are all different)

Expressions

An expression is a mathematical "phrase" that stands for a single number.
An equation consists of two expressions connected by an equals sign.
It can only be true or false.

Example:
a number less than five
5 - x
five less than a number
x – 5

Equations

An equation is a mathematical "sentence" that says that two things are equal.
An expression is never true or false, but just has a numerical value.

Example:
Ten is five less than a number.
10 = x – 5

A number is less than five.
x < 5

Linear Equations

Solving a Linear Equation : Get the variable you are solving for alone on one side and everything else on the other side using INVERSE operations.
Example:
$x - 5 = 2$
$x - 5 + 5 = 2 + 5$
$x = 7$

$y + 4 = -7$

$y + 4 - 4 = -7 - 4$

$y = -11$

$5x = 7$
$\frac{5x}{5} = \frac{7}{5}$
$x = \frac{7}{5}$

$\frac{x}{2} = 5$
$(2)\frac{x}{2} = (2)5$
$x = 10$

Unit 3: Geometric Applications of Exponents

Volume of a Cylinder:

V = Bh **$V = \pi r^2 h$**

(B = area of base: πr^2)

(h = height of the figure)

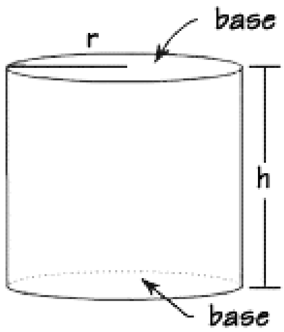
A Coke can is 5 inches tall and has a radius of 2 inches. What is the volume of the can?

L: r = 2 h = 5

W: $V = \pi r^2 h$

P: $V = (3.14)(2^2)(5)$

C: $V = 62.8 \text{ in}^3$



Volume of a Cone:

$V = \frac{Bh}{3}$ $V = \frac{\pi r^2 h}{3}$

(B = area of base: πr^2)

($\pi \approx 3.14$)

(r = radius)

(h = height of the figure)

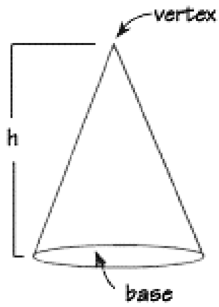
What is the volume of an ice cream cone with a radius of 3 and height of 4?

L: r = 3 h = 4

W: $V = \frac{\pi r^2 h}{3}$

P: $V = \frac{(3.14)(3^2)(4)}{3}$

C: $V = 37.7 \text{ units}^3$



Volume of a Sphere:

$V = \frac{4\pi r^3}{3}$

($\pi \approx 3.14$)

(r = radius)

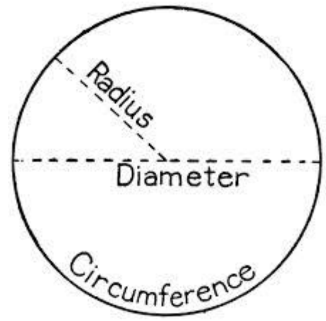
You are playing softball with friends. The ball has a diameter of 10 cm. What is the volume of the softball?

L: d = 10 r = 5

W: $V = \frac{4\pi r^3}{3}$

P: $V = \frac{(4)(3.14)(5^3)}{3}$

C: $V = 523.3 \text{ cm}^3$



Label the information
Write the formula
Plug in the information
Chug out the answer

The **diameter** of a circle is longest distance across a circle. (The diameter cuts through the center of the circle. This is what makes it the longest distance.)

The **radius** of a circle is the distance from the center of the circle to the outside edge.

Pythagorean Theorem:

*only works with right triangles

(right triangles have only 1 right angle)

$\mathbf{a^2 + b^2 = c^2}$

a & b are legs

c is hypotenuse, longest side, opposite right angle

The Pythagorean theorem: The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c).

To find the missing leg: Take the area of the hypotenuse (area of the square on the hypotenuse) – the leg squares (area of the square on the leg) = the leg squared (area of the square on the leg).

To find the length of the leg: take the square root of the area of the square on the leg.

To find the missing hypotenuse: take the leg squared (area of the square on the leg) + the leg squared (area of the square on the leg) = the hypotenuse squared (area of the square on the hypotenuse).

To find the length of the hypotenuse: take the square root of the area of the square on the hypotenuse.

You are creating a picture frame in the shape of a right triangle. You have calculated the longest side to be 15 inches. What would be the length of the other two sides?

$a^2 + b^2 = 15^2$

$a^2 + b^2 = 225$

$\mathbf{9}^2 + \mathbf{12}^2 = 225$

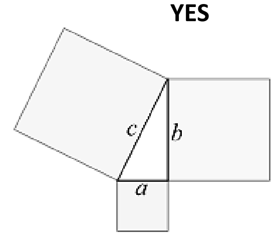
$81 + 144 = 225$

The other two sides are 9 in. and 12 in.

Is it a Right Triangle?

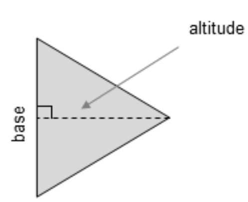
Plug into $a^2 + b^2 = c^2$ (c is biggest #)

4, 6, 8	**3, 4, 5**
$4^2 + 6^2 = 8^2$	$3^2 + 4^2 = 5^2$
$16 + 36 = 64$	$9 + 16 = 25$
$52 \neq 64$	$25 = 25$
NO	**YES**



The Altitude of a triangle

In the case of a triangle, a common way to calculate its area is 'half base times height' where the 'height' is the altitude, or the perpendicular distance from the base to the opposite vertex. The base can be any side, not just the one drawn at the bottom.



To calculate the area you pick one side to be the base, and then measure the altitude at right angles to it.

Converse of Pythagorean Theorem:

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

Length of leg/hypotenuse measured in: ft, cm, m, etc.

Area of square on leg/hypotenuse measured in:
ft^2, cm^2, m^2, etc.

Literal Equations

Equations with multiple variables where you are asked to solve for just one of the variables.

Example:

Solve $V = \pi r^2 h$ for r.

Step 1: Divide both sides by π and h

$\frac{V}{\pi h} = r^2$

Step 2: Take the square root of both side to get r by itself

$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2}$

Step 3: Solve for r.

$\sqrt{\frac{V}{\pi h}} = r$

Example:

Solve $V = \pi r^2 h$ for h.

Step 1: To solve for h, you will need to divide both sides by π and r^2.

$\frac{V}{\pi r^2} = h$

Example:

The formula for finding the perimeter of a rectangle is P = 2b + 2h. Solve for h.

Step 1: subtract 2b from each side

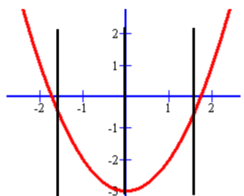
P – 2b = 2h

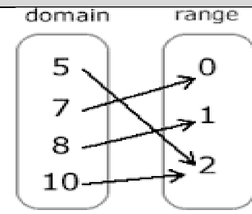
Step 2: Divide both sides by 2

$\frac{P - 2b}{2} = h$

Unit 4: Functions

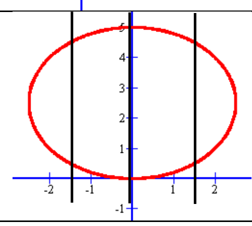
Function –
one output for every input (x values can't repeat, pass vertical line test)

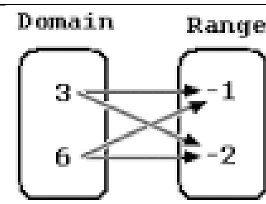




x	y
2	8
3	9
5	10
4	11

Not a Function –
Doesn't pass vertical line test, x values repeat…

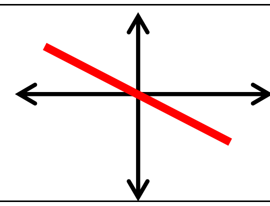




x	y
1	3
2	2
2	10
3	4

Linear –
Have to have a common difference, have the slope intercept form (y=mx+b), and form a straight line when graphed.

x	1	2	3	4
y	3	6	9	12

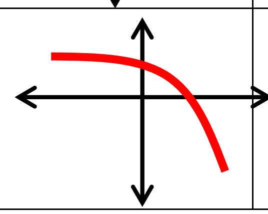


y = 2x + 1

y = -9x - 4

Nonlinear –
Are a curved or broken line when graphed; in the equation there are exponents, variables multiplied together, or variables in the denominator.

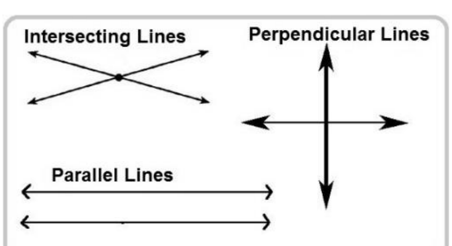
x	5	6	7	8
y	2	4	7	8

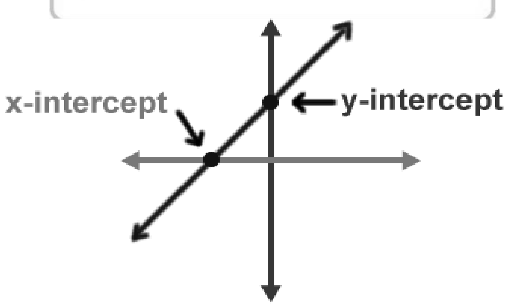


$y = x^3$

$8 = 6xy$

$\mathbf{3 = \frac{3x}{y}}$

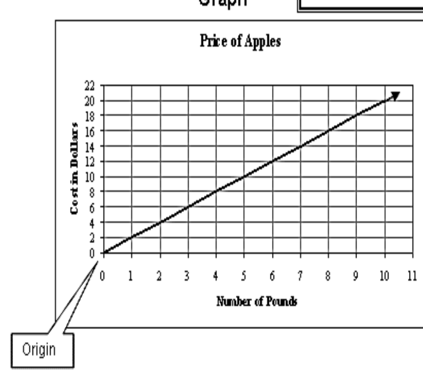




Proportional Relationships

Apples for Sale
$2.00 per pound

Graph



Table

Price of Apples	
Number of Pounds of Apples (x)	Cost (y)
0	0
1	$2.00
2	$4.00
3	$6.00
4	$8.00
5	$10.00

In this example, to find the cost, the number of pounds is always multiplied by $2.00 which is the constant rate of change (k).

Unit 5: Linear Functions

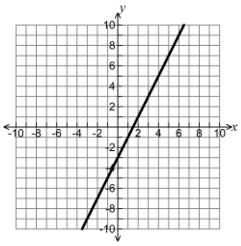
To find an equation, you always need a slope (m) and y-intercept (b)!!!!!

Equation From a Graph

1. Find the y-intercept (**b**)
2. Locate another point
3. From the "b" use rise over run to get to the next point – this is your slope (**m**)
4. Put "m" and "b" into

   y = **m**x + **b**

   y = 2x -3



Equation From a Table

1. Find $\frac{\text{change in y}}{\text{change in x}}$ = ***m***
2. Pick a point (x,y) and plug into y=mx+b along with ***m***
3. Solve for "*b*"
4. Put "*m*" and "*b*" into

   y = ***m***x + ***b***

x	*y*
-1	-2
0	2
1	6

Difference in *x* is

Difference in *y* is +4

If x is 0, then this is your y-intercept

m = $\frac{4}{1}$ **b** = (0, 2) = 2

y = 4x + 2

Equation From (x_1,y_1) (x_2,y_2)

1. Find $\frac{y_2 - y_1}{x_2 - x_1}$ = ***m***
2. Pick a point (x,y) and plug into y = *m*x+*b* along with ***m***
3. Solve for "*b*"
4. Put "*m*" and "*b*" into

   y = ***m***x + ***b***

Determine equation from points (0, -4) and (0, 5).

$\frac{5 - -4}{0 - 0} = \frac{5+4}{0} = \frac{9}{0}$ = undefined slope

(remember equation is *x* =)

x = 0

Standard Form

ax + by = c

Convert to slope intercept form

move ax to opposite side with opposite sign (by = -ax + c)

1. *y* stands alone; divide everything by *b*

   (y = $-\frac{a}{b}$x + $\frac{c}{b}$)

2. *y* cannot be negative (change everything in equation to opposite sign (multiplying by -1))

Word Problem

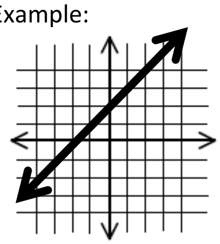
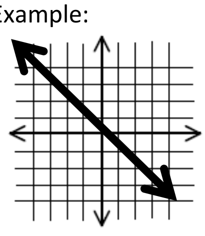
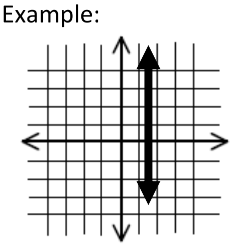
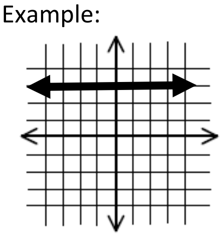
The distance traveled on a trip is directly proportional to the speed of the car. A car traveled 300 miles in six hours. Write an equation to represent *y*, the distance the car would travel in *x* hours.

REMEMBER: Unit Rate = SLOPE

$\frac{300}{6}$ = 50

y = 50x

Types of SLOPE

POSITIVE SLOPE	NEGATIVE SLOPE	UNDEFINED SLOPE	ZERO SLOPE
Graphed line moves **<u>upward</u>** from left to right.	Graphed line moves **<u>downward</u>** from left to right.	Graphed line is a **<u>vertical line</u>** (straight up and down).	Graphed line is a **<u>horizontal line</u>**.
Example: 	Example: 	Example: 	Example: 
Example: y = 6x + 1 y = $\frac{2}{3}$x – 4	Example: y = -3x + 2 y = $-\frac{2}{5}$x - 1	Example: x = 4 x = -6 x = 3	Example: y = 5 y = -2 y = 8

Unit 6: Linear Models & Tables

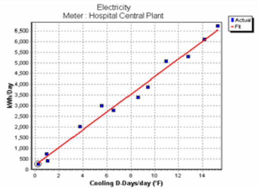
Rate of Change

Increasing – positive slope	Decreasing – negative slope	Greatest ROC= Ignore the sign (doesn't matter if positive or negative) and choose biggest number	Least ROC = Ignore the sign (doesn't matter if positive or negative) and choose smallest number

Stories from Graphs

Going away from = distance increasing = positive slope	Going towards = distance decreasing = negative slope	Running = steeper line	Walking = less steep line

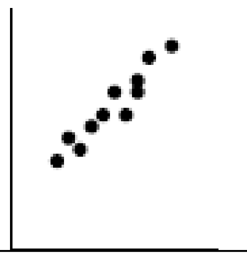
Line of Best Fit

1. Put your ruler in the middle of as many points as possible
2. Draw a straight line across whole graph (This line may pass through some of the points, none of the points, or all of the points).


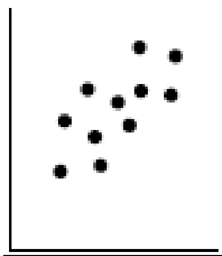
3. Find your "b" - Look at where the line crosses the y-axis
4. Find your slope = pick two points and count rise over run.

Scatter Plots

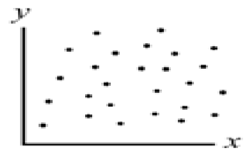
Strong Correlation – close together
Weak Correlation – far apart



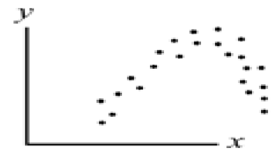
Strong Positive Correlation



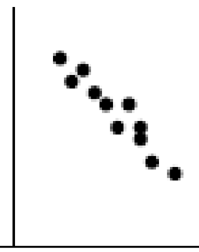
Weak Positive Correlation



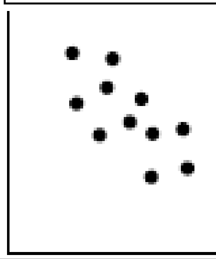
no correlation



non linear

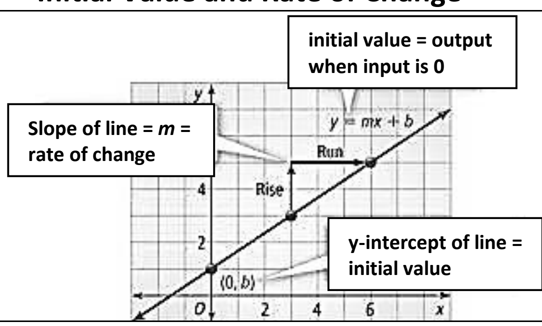


Strong Negative Correlation



Weak Negative Correlation

Initial Value and Rate of Change



Joint and Marginal Frequencies

Joint Frequencies

	Spanish	French	German	Total
Boys	10	2	8	20
Girls	15	12	3	30
Total	25	14	11	50

Marginal Frequencies

Two-Way Tables and Relative Frequency Tables

Two-Way Frequency Table

	Spanish	French	German	Total
Boys	10	2	8	20
Girls	15	12	3	30
Total	25	14	11	50

Divide all table entries by the table total (50)

Two-Way Relative Frequency Table (with respect to table total)

	Spanish	French	German	Total
Boys	0.2	.04	0.16	0.40
Girls	0.3	0.24	0.06	0.60
Total	0.5	0.28	0.22	1.00

Variables

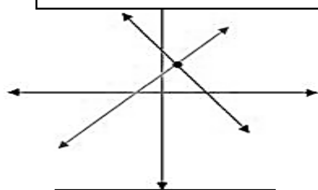
Variables can be classified as **categorical** (aka, qualitative) or **quantitative** (aka, numerical).

- Categorical. Categorical variables take on values that are names or labels. The color of a ball (e.g., red, green, blue) or the breed of a dog (e.g., collie, shepherd, terrier) would be examples of categorical variables.
- Quantitative. Quantitative variables are numerical. They represent a measurable quantity. For example, when we speak of the population of a city, we are talking about the number of people in the city - a measurable attribute of the city. Therefore, population would be a quantitative variable
- An outlier is an observation that lies outside the overall pattern of a distribution. Usually, the presence of an outlier indicates some sort of problem. This can be a case which does not fit the model under study, or an error in measurement.

Unit 7: Systems of Equations

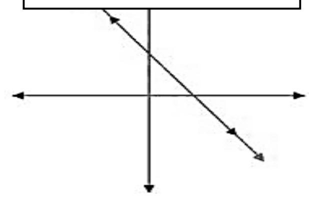
Solution: (x,y)
Where the lines intersect

Consistent and Independent



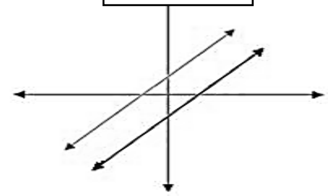
Intersecting Lines
Exactly one solution

Consistent and Dependent



Same Lines
Infinitely many solutions

Inconsistent



Parallel Lines
No solution

3 Methods to Solving Systems:

GRAPHING
-Both equations must be written as $y = mx+b$!
-Graph both the lines and see where they meet!

SUBSTITUTION
-Single out *x* or *y* and then plugging this back into the other equation, "substituting" for the chosen variable and solving for the other.
-Then you back-solve for the first variable.

ELIMINATION
-Line them up.
-Look for matching coefficients.
-Add/subtract to eliminate.

Elimination

$$\begin{cases} 3y + 2x = 6 \\ 5y - 2x = 10 \end{cases}$$

Eliminate the x-variable by addition of the two equations.

$3y + 2x = 6$
$+\ 5y - 2x = 10$
$= 8y \quad = 16$
$y \quad = 2$

The value of y can now be substituted into either of the original equations to find the value of x.

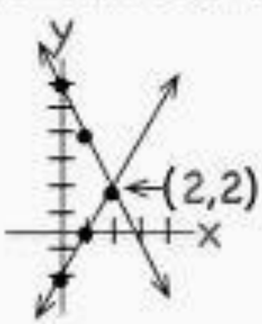
$3y + 2x = 6$
$3\cdot 2 + 2x = 6$
$6 + 2x = 6$

Graphing

Find the Solution

$2x - y = 2$
$y = 2x - 2$

$2x + y = 6$
$y = -2x + 6$



$2x - y = 2$
$2(2) - 2 \stackrel{?}{=} 2$
$4 - 2 \stackrel{?}{=} 2$
✔ $2 = 2$

Solution: (2,2)

Substitution

$y = x + 1 \qquad 2y = 3x$

$2y = 3x$
$2(x + 1) = 3x$
$2x + 2 = 3x$
$-2x \qquad -2x$
$2 = x$

$y = x + 1$
$y = 2 + 1 = 3$

Solution: (1, 3)